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A time delay control for a nonlinear dynamic beam under moving load

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Abstract

The bifurcation resulted from moving force may lead to instability for the system. Based on time delay feedback controller, a nonlinear beam under moving load is discussed in the case of the primary resonance and the 1/3 subharmonic resonance. The bifurcation may be eliminated or the bifurcation point's position may be changed. The perturbation method is used to obtain the bifurcation equation of the nonlinear dynamic system. The result indicates time delay feedback controller may work well on this system, but the selection of a proper time delay and its coefficient may depend on the engineering condition. This paper presents some theoretical results.

1. Introduction

Beams are widely used in machines, architectural structures and aircrafts. The dynamic behavior becomes more complicated due to edging operating environments and more severe dynamic loads. Therefore, proper active control is essential in order to ensure the beam structure stays will constrained. Besides, time delay always exists in feedback control loop due to sensing or computation process. The problem of the system's dynamic stability, which is caused by the time delay effect, has made the system more controllable and unstable. Thus, the issue of how to enhance the system's robustness to the time delay effect is worth further study. In order to obtain a proper control force exerted on the system, accurate and optimal computation is necessary. However, it will cause time delay in the system in the actuating force. The results will not match the system's need on real time. Therefore, the need for a methodology that is simple in calculation and effective in control is critical.

During the 20th century, many studies were carried out to analyze the dynamics of structures under moving loads. The interest was originally oriented toward bridges and railways to study the conditions under which these structures are stable [1,2]. The principal results show that compared to a beam under a static load, the load inertia modifies the beam dynamics in two ways: the inertia renders the beam deflection higher, and resonance is reached at a lower moving load velocity [3,4]. Dugush and Eisenberger [5] applied modal analysis

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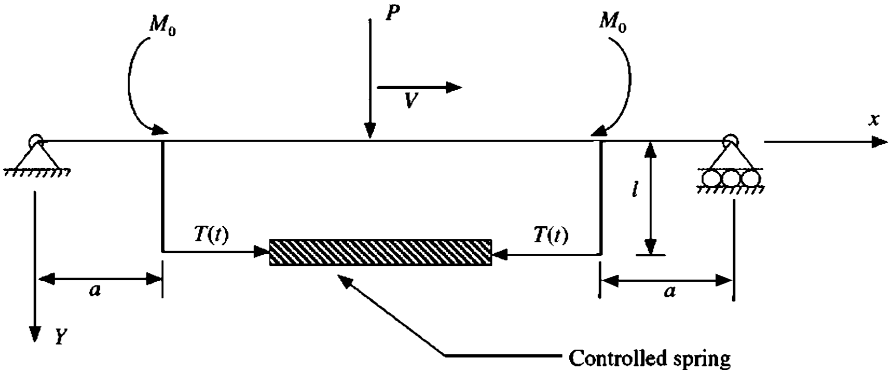

Fig. 1. Dynamic beam structure.

in combination with integral transformation methods to determine the dynamic deflection and the internal forces of a multi-span non-uniform beam under moving loads. They showed that a small number of mode shapes are required to obtain accurate solutions. Zhu and Law [6] studied the dynamic loading on a multi-lane continuous bridge due to vehicles moving on top of the bridge deck and highlighted the influence of the transverse vehicle position and road surface roughness on the dynamic impact factor, defined as the ratio of the maximum dynamic response to the maximum static response. Gbadeyan and Oni [7] developed a general approach to determine solutions of both moving force and moving mass problems for both Euler–Bernoulli and Rayleigh beams having any of the classical end-support conditions.

Recently, stabilization of systems with time delay has received considerable attention. Several linear state feedback controllers have been proposed by Su [8], Chou [9] and Shen [10]. The fundamental designs are based on (1) pole placement approach, (2) Lyapunov approach, and (3) linear quadratic regulator approach. In these cases, time delay can be the source of instability. Basharkhah and Yao [11] found that time delay could make the control system lose its reliability. In 1985, Abdel-Roham [12] applied the pole placement method to compensate for the system's time delay. Lo'pez-Almansa and Rodellar [13] applied independent mode space control for predictive control, and their experiments showed that when the number of sensors and actuators is less than the system's mode number, the control system becomes unstable if the time delay effect is taken into account.

In this paper, a dynamic beam structure under active control by a servomechanism is shown in Fig. 1.

For this nonlinear dynamic system, moving fore may lead to bifurcation and the bifurcation resulted from moving force may lead to instability for the system. So studying the bifurcation control is meaningful. In this paper, two cases, the primary resonance and the 1/3 subharmonic resonance are discussed theoretically.

2. Equations of motion

In discussing beams, attention is restricted to planar and non-rotating motions. The equations of motion are obtained by a combination of the inertial effect of the moving load and the nonlinear effect in the beam dynamics [14]. Assuming that plane sections remain plane and material meets linear stress–strain law, the equations of motion governing the nonlinear dynamics of a beam with uniform shape subjected to a moving load P and velocity v are given by

$$\rho S u_{tt} - ES u_{xx} = \frac{1}{2}(ES)\frac{\partial}{\partial x}\left[(1-2u_x)y_x^2\right], \tag{1}$$

$$\rho S y_{tt} + EI y_{xxxx} + c y_t = P\delta(x - vt) + ES\frac{\partial}{\partial x}(e y_x) + M_0\delta'(x-a) - M_0\delta'(x-L+a), \tag{2}$$

where $e = u_x - u_x^2 + 1/2y_x^2$, E is Young's modulus of the beam, ρ is the beam density, S and I denote the area and inertia moment the beam cross-section, respectively. $y(x, t)$ is the vertical deflection of the beam, while u is the axial displacement, y and u depend on the spatial coordinate x and the time t. δ is Dirac delta function and δ' is the derivative. Control torque M_0 is exerted by the servomechanism, installed beneath the central portion of the beam at a distance a measured from both end supports, which tends to balance the bending phenomenon or control the nonlinear dynamic characters caused by the moving load. When the servomechanism is in action, the actuator will increase or decrease the spring displacement according to the control system's needs. The terms $M_0\delta'(x-a) - M_0\delta'(x-L+a)$ are the control forces produced by servomechanism. The active control torque M_0 is designated as

$$M_0 = lK\Delta(t) = Kl\left[u(t) + l\frac{\partial y(a,t)}{\partial x} - l\frac{\partial y(L-a,t)}{\partial x}\right], \tag{4}$$

where K is the stiffness of the spring, $u(t)$ is the spring displacement caused by servomechanism and $\Delta(t)$ is the displacement of the spring. Where $u(t) = 0$ in Eq. (4) correspond to a passive structure control system. Assuming that the longitudinal inertial terms u_{tt} and u_x^2 are negligible and using the assumed mode methods, Eqs. (1) and (2) can be reduced to

$$\ddot{Y}_j(t) + 2\xi\omega_j\dot{Y}_j(t) + \omega_j^2 Y(t) = \begin{cases} \frac{2P}{mL}\sin j\omega t + \frac{j^2\pi^4 r^2}{L^4} Y_j \sum\limits_{k=1}^{\infty} Y_k^2 k^2, & j = 2, 4, \ldots \\ \frac{2P}{mL}\sin j\omega t + \frac{j^2\pi^4 r^2}{L^4} Y_j \sum\limits_{k=1}^{\infty} Y_k^2 k^2 - \frac{4j\pi M_0(t)}{mL^2}\sin\frac{j\pi a}{L}, & j = 1, 3, \ldots \end{cases} \tag{5}$$

where $Y_j(t) = (2/L)\int_0^L y(x,t)\sin(j\pi x/L)\mathrm{d}x$ is the beam displacement of the jth mode, ξ is the damping ratio, $\omega_j = (j\pi/L)^2\sqrt{EI/m}$ is the natural angular frequency of the jth mode, $\omega = \pi v/L$ is the natural angular frequency, $m = \rho S$ and

$$M_0(t) = Kl\left[u(t) + 2L\sum_{j=1,3,\ldots}^{\infty}\frac{j\pi}{L}\cos\frac{j\pi a}{L}Y_j(t)\right]. \tag{6}$$

Since the high order modes of motion contribute little to bending displacement and the nonlinear characters, only the basic mode is considered. Then the equation of motion can be written as

$$\ddot{Y}(t) + \omega_0^2 Y(t) + \varepsilon\mu\dot{Y}(t) = \varepsilon f\,\sin\,\omega t + \varepsilon\alpha Y^3(t) + \varepsilon p u(t), \tag{7}$$

where

$$\omega_0^2 = \omega_1^2 - 2\pi Kl\,\cos\frac{\pi a}{L}, \quad \varepsilon\mu = 2\xi\omega_1, \quad \varepsilon f = \frac{2P}{mL}, \quad \varepsilon\alpha = \frac{\pi^4 r^2}{L^4}, \quad r^2 = \frac{E}{S}.$$

In Eq. (7), $u(t)$ is the active control function, which can be designated or be designed for needs. When the function is designed as displacement time-delay feedback control, the simplest form can be written as $u(t) = Y(t-\tau)$. Where τ is the time delay, $p > 0$ corresponding to positive feedback and $p < 0$ corresponding to negative feedback. So the equation of motion with displacement feedback can be written as

$$\ddot{Y}(t) + \omega_0^2 Y(t) + \varepsilon\mu\dot{Y}(t) = \varepsilon f\,\sin\,\omega t + \varepsilon\alpha Y^3(t) + \varepsilon p Y(t-\tau). \tag{8}$$

In the following sections, the primary and 1/3 subharmonic resonances will be, respectively, studied by using the method of multiple scales.

3. Primary resonance

By using the method of multiple scales [15–17], the perturbation solution of Eq. (8) is assumed as follows:

$$Y = Y_0(T_0, T_1) + \varepsilon Y_1(T_0, T_1) + \ldots, \tag{9}$$

where $T_0 = t$, $T_1 = \varepsilon t$. In the case of primary resonance, we let

$$\omega_0 = \omega + \varepsilon\sigma. \tag{10}$$

Hence $\omega_0^2 = \omega^2 + 2\varepsilon\omega\sigma + \varepsilon^2\sigma^2$, σ is the detuning parameter. Substituting Eqs. (9) and (10) into Eq. (8) and equating coefficients of like powers of ε yield the following equations:

$$D_0^2 Y_0 + \omega^2 Y_0 = 0, \tag{11}$$

$$D_0^2 Y_1 + \omega^2 Y_1 = -2D_0 D_1 Y_0 - 2\omega\sigma Y_0 - \mu D_0 Y_0 + \alpha Y_0^3 + f\ \sin\omega T_0 + pY_0(t-\tau), \tag{12}$$

The solution of Eq. (11) is written as follows:

$$Y_0 = A(T_1)\mathrm{e}^{\mathrm{i}\omega T_0} + \bar{A}(T_1)\mathrm{e}^{-\mathrm{i}\omega T_0}. \tag{13}$$

Substituting Eq. (13) into Eq. (12) we obtain

$$D_0^2 Y_1 + \omega^2 Y_1 = \left[-2\mathrm{i}\omega D_1 A - 2\omega\sigma A - \mathrm{i}\mu\omega A + 3\alpha A^2\bar{A} - \frac{f}{2}\mathrm{i} + p(\cos\omega\tau - \mathrm{i}\ \sin\omega\tau)A\right]\mathrm{e}^{\mathrm{i}\omega T_0} + \alpha A^3\mathrm{e}^{3\mathrm{i}\omega T_0} + \mathrm{cc}. \tag{14}$$

Eliminating secular terms from Eq. (14), we have

$$-2\mathrm{i}\omega D_1 A - 2\omega\sigma A - \mathrm{i}\mu\omega A + 3\alpha A^2\bar{A} - \frac{f}{2}\mathrm{i} + p(\cos\omega\tau - \mathrm{i}\ \sin\omega\tau)A = 0. \tag{15}$$

Let $A = a\mathrm{e}^{\mathrm{i}\varphi}$. Substituting it into Eq. (15) and separating the real and imaginary part yields the averaged equation as follows:

$$\begin{aligned}\frac{\mathrm{d}a}{\mathrm{d}T_1} &= -\frac{1}{2}\left(\mu + \frac{p}{\omega}\sin\omega\tau\right)a - \frac{1}{4\omega}f\ \cos\varphi,\\ a\frac{\mathrm{d}\phi}{\mathrm{d}T_1} &= \left(\sigma - \frac{p}{2\omega}\cos\omega\tau\right)a - \frac{3\alpha}{2\omega}a^3 + \frac{1}{4\omega}f\ \sin\varphi.\end{aligned} \tag{16}$$

For the case of $\mathrm{d}a/\mathrm{d}T_1 = \mathrm{d}\varphi/\mathrm{d}T_1 = 0$, there is steady-state response in the system (8). The result is

$$\begin{aligned}&-\frac{1}{2}\left(\mu + \frac{p}{\omega}\sin\omega\tau\right)a - \frac{1}{4\omega}f\ \cos\varphi = 0,\\ &\left(\sigma - \frac{p}{2\omega}\cos\omega\tau\right)a - \frac{3\alpha}{2\omega}a^3 + \frac{1}{4\omega}f\ \sin\varphi = 0.\end{aligned} \tag{17}$$

Eliminating φ from Eq. (17), we obtain the bifurcation equation

$$\frac{1}{4}\mu_e^2 a^2 + \left(\sigma_e - \frac{3\alpha}{2\omega}a^2\right)^2 a^2 = \frac{1}{16\omega^2}f^2, \tag{18}$$

where

$$\mu_e = \mu + \frac{p}{\omega}\sin\omega\tau, \quad \sigma_e = \sigma - \frac{p}{2\omega}\cos\omega\tau. \tag{19}$$

Obviously, the amplitude of the response is a function of the external detuning, feedback with time delay and the amplitude of the excitation.

The peak amplitude of the primary resonance, obtained from Eq. (18), is given by

$$a_p = \frac{f}{2|\mu_e|\omega}. \tag{20}$$

The real solution a of Eq. (18) determines the primary resonance response amplitude. There can be either one or three real solutions. Three real solutions exist between two points of vertical tangents (saddle-node bifurcation), which are determined by differentiation of Eq. (18) implicitly with respect to a^2.

This leads to the condition

$$\sigma_e^2 - \frac{6}{\omega}\alpha a^2 \sigma_e + \frac{27}{4\omega^2}\alpha^2 a^4 + \frac{1}{4}\mu_e^2 = 0. \tag{21}$$

With solutions

$$\sigma_e^{\pm} = \frac{3\alpha}{\omega}a^2 \pm \frac{1}{2}\left(\frac{9}{\omega^2}\alpha^2 a^4 - \mu_e^2\right)^{1/2}. \tag{22}$$

For the case $(9/\omega^2)\alpha^2 a^4 > \mu_e^2$, there exists an interval $\sigma^- < \sigma_e < \sigma^+$ in which three real and positive solutions a of Eq. (18) exist. In the limit $(9/\omega^2)\alpha^2 a^4 \to \mu_e^2$ this interval shrinks to the point $\sigma_e = (3/\omega)\alpha a^2$. The critical force amplitude obtained from Eq. (18) is

$$f_{\text{crit}} = 2\omega\mu_e(5\omega\mu_e/3\alpha)^{1/2}. \tag{23}$$

For $f < f_{\text{crit}}$ there is only one solution while for $f > f_{\text{crit}}$ there are three. The stability of the solutions is determined by the eigenvalues of the corresponding Jacobian matrix of Eq. (16). The corresponding eigenvalues are the root of

$$\lambda^2 + \mu_e\lambda + \frac{1}{4}\mu_e^2 + \left(\sigma_e - \frac{3}{2\omega}\alpha a^2\right)\left(\sigma_e - \frac{9}{2\omega}\alpha a^2\right) = 0. \tag{24}$$

It turns out that the sum of the two eigenvalues is $-\mu_e$. For the uncontrolled system, the sum of two eigenvalues is $-\mu$, which is negative. The addition of the feedback gains and time-delays varies the sum of the two eigenvalues. Three cases such as $\mu_e > 0$, $\mu_e = 0$ and $\mu_e < 0$ may occur depending on the values of the feedback with time-delays. If the feedback with time-delays are chosen in such a way that the sum of the two eigenvalues is positive ($\mu_e < 0$), at least one of the eigenvalues will always have a positive real part. The system will be unstable. The selection of the feedback with time-delays is not possible. On the other hand, if the sum of the two eigenvalues is zero ($\mu_e = 0$) by a certain value of the feedback with time-delays, a pair of purely imaginary eigenvalues and hence a Hopf bifurcation may occur. Anyhow, the above two cases should be avoided from the viewpoint of bifurcation control. The feedback should be implemented at least in such a way that $\mu_e > 0$ is guaranteed. Under such feedback gains and time-delays, the sum of two eigenvalues is always negative, and accordingly, at least one of the two eigenvalues will always have a negative real part. The other eigenvalues is zero when

$$\mu_e^2 + \left(\sigma_e + \frac{3}{2\omega}\alpha a^2\right)\left(\sigma_e + \frac{9}{2\omega}\alpha a^2\right) = 0. \tag{25}$$

Based on the before-mentioned analyses, the sufficient condition of guaranteeing the system stability is letting μ_e satisfying

$$\frac{9}{\omega^2}\alpha a^4 - \mu_e^2 < 0, \quad \mu_e > 0. \tag{26}$$

As a case, the parameters μ, ω, α are confirmed as $\mu = 0.2$, $\omega = 1$, $\alpha = 0.4$. The curves relate with time delay are obtained and figured in Fig. 2.

In Fig. 2(a), assumed amplitude of excitation $f = 0.1$, and in Fig. 2(b), assumed detuning parameter $\sigma = 0$. The thin line is corresponding to origin system and dashed line corresponding controlled system with time delay, time delay $\tau = \pi/2$. Obviously, when τ is assumed some values, the bifurcation can be eliminated or the bifurcation point be changed. Fig. 2(c) and (d) are corresponding to the critical force amplitude and the peak of the primary resonance changing with time delay τ. Fig. 2(c) and (d) indicate that the change rule of the critical force amplitude and the peak of the primary resonance is not whole conformable. So the proper value of τ may take into account the requirement of engineering.

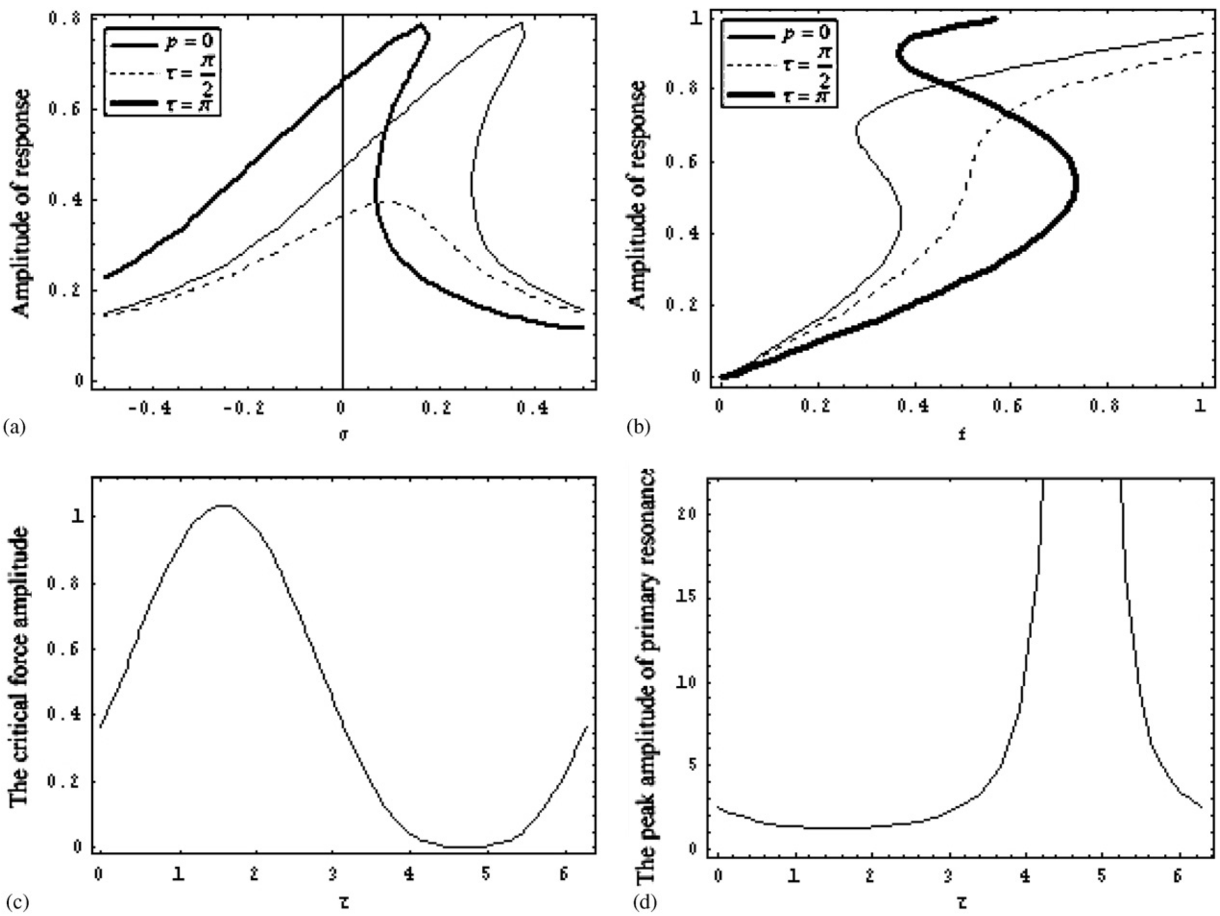

Fig. 2. The curves relate with time delay.

4. Subharmonic resonance

For the case of subharmonic resonance, it is assumed that

$$F = \varepsilon f, \quad \omega_0 = \frac{1}{3}\omega + \varepsilon\sigma. \tag{27}$$

Substituting Eqs. (9) and (27) into Eq. (8) and equating coefficients of like powers of ε yield the following equations:

$$D_0^2 Y_0 + \frac{1}{9}\omega^2 Y_0 = F \sin \omega T_0, \tag{28}$$

$$D_0^2 Y_1 + \frac{1}{9}\omega^2 Y_1 = -2D_0 D_1 Y_0 - \frac{2}{3}\omega\sigma Y_0 - \mu D_0 Y_0 + \alpha Y_0^3 + p Y_0(t-\tau). \tag{29}$$

The solution of Eq. (28) is written as follows:

$$Y_0 = A(T_1)\mathrm{e}^{\mathrm{i}(1/3)\omega T_0} - \frac{9}{16}F\mathrm{i}\mathrm{e}^{\mathrm{i}\omega T_0} + \mathrm{cc}. \tag{30}$$

Substituting Eq. (30) into Eq. (29) we obtain

$$D_0^2 Y_1 + \frac{1}{9}\omega^2 Y_1 = \left[-\frac{2}{3}\mathrm{i}\omega D_1 A - \frac{2}{3}\omega\sigma A - \frac{\mathrm{i}}{3}\mu\omega A + 3\alpha A^2\bar{A} - \frac{243}{128}\alpha A F^2 - \frac{27}{16}\mathrm{i}\alpha\bar{A}^2 F - p\left(\cos\frac{1}{3}\omega\tau - \mathrm{i}\sin\frac{1}{3}\omega\tau\right)A\right]\mathrm{e}^{\mathrm{i}/3\omega T_0} + \mathrm{NST} + \mathrm{cc}. \tag{31}$$

Eliminating secular terms from Eq. (31), we have

$$-\frac{2}{3}\mathrm{i}\omega D_1 A - \frac{2}{3}\omega\sigma A - \frac{\mathrm{i}}{3}\mu\omega A + 3\alpha A^2\bar{A} - \frac{243}{128}\alpha A F^2 - \frac{27}{16}\mathrm{i}\alpha\bar{A}^2 F - p\left(\cos\frac{1}{3}\omega\tau - \mathrm{i}\sin\frac{1}{3}\omega\tau\right)A = 0. \tag{32}$$

Let $A = a\mathrm{e}^{\mathrm{i}\varphi}$. Substituting it into Eq. (32) and separating the real and imaginary part yield the average equation as follows:

$$\frac{\mathrm{d}a}{\mathrm{d}T_1} = -\frac{1}{2}\left(\mu + \frac{3p}{\omega}\sin\frac{1}{3}\omega\tau\right)a - \frac{81}{32}\alpha F a^2\cos 3\varphi,$$
$$a\frac{\mathrm{d}\phi}{\mathrm{d}T_1} = \left(\sigma - \frac{3p}{2\omega}\cos\omega\tau\right)a + \frac{729}{256}\alpha F^2 a - \frac{9\alpha}{2\omega}a^3 - \frac{81}{32}\alpha F a^2\sin 3\varphi. \tag{33}$$

For the case of $\mathrm{d}a/\mathrm{d}T_1 = \mathrm{d}\varphi/\mathrm{d}T_1 = 0$, there is steady-state subharmonic resonance response in the system (8). The result is

$$-\frac{1}{2}\left(\mu + \frac{3p}{\omega}\sin\frac{1}{3}\omega\tau\right)a - \frac{81}{32}\alpha F a^2\cos\varphi = 0,$$
$$\left(\sigma - \frac{3p}{2\omega}\cos\omega\tau\right)a + \frac{729}{256}\alpha F^2 a - \frac{9\alpha}{2\omega}a^3 - \frac{81}{32}\alpha F a^2\sin\varphi = 0. \tag{34}$$

Eliminating φ from Eq. (34), we obtain the bifurcation equation

$$\frac{1}{4}\mu_e^2 a^2 + \left(\sigma_e + \frac{729}{256}\alpha F^2 - \frac{9\alpha}{2\omega}a^2\right)^2 a^2 = \left(\frac{81}{32}\alpha F\right)^2 a^4, \tag{35}$$

where

$$\mu_e = \mu + \frac{3p}{\omega}\sin\frac{1}{3}\omega\tau, \quad \sigma_e = \sigma - \frac{3p}{2\omega}\cos\frac{1}{3}\omega\tau. \tag{36}$$

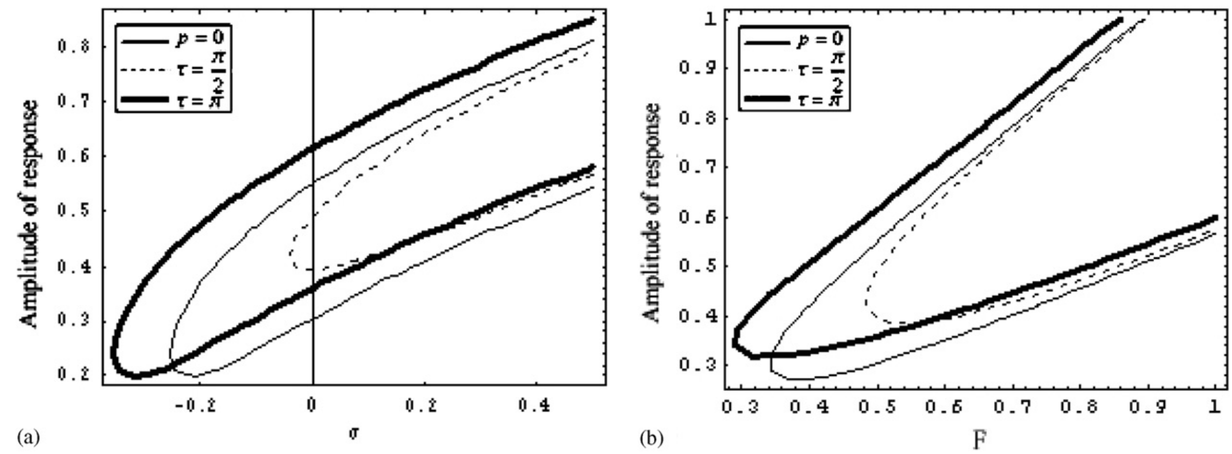

Fig. 3. The curves of subharmonic bifurcation.

There are two possibilities: either a trivial solution $a = 0$, or non-trivial solutions, which are given by

$$\frac{1}{4}\mu_e^2 + \left(\sigma_e + \frac{729}{256}\alpha F^2 - \frac{9\alpha}{2\omega}a^2\right)^2 = \left(\frac{81}{32}\alpha F\right)^2 a^2. \tag{37}$$

The steady-state solutions of subharmonic resonance response is determined by the eigenvalues of the characteristic equation, which are the roots of

$$\lambda^2 + \mu_e\lambda + \frac{1}{4}\mu_e^2 + \left(\sigma_e + \frac{729}{256}\alpha F^2 - \frac{9}{2\omega}\alpha a^2\right)\left(\sigma_e + \frac{729}{256}\alpha F^2 - \frac{27}{2\omega}\alpha a^2\right) = 0. \tag{38}$$

As a case, the curves of Fig. 3 are corresponding to the system with parameters $\mu = 0.2$, $\omega = 3$, $\alpha = 0.4$. Fig. 3 indicates that the subharmonic bifurcation cannot be eliminated by changing time delay. For this case, we can only chose the proper value of τ to change the bifurcation point's position.

5. Conclusion

The motion equation for governing the nonlinear dynamics of a beam with uniform shape subjected to a moving load and velocity is set up. The bifurcation resulted from moving force may lead to instability for the system. Based on time delay feedback controller, the bifurcation may be eliminated or the bifurcation point's position may be changed. The result about the primary resonance and the 1/3 subharmonic resonance indicates time delay feedback controller may work well on this system, but the detailed chosen for proper time delay and its coefficient may reckon on the engineering requirement.

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